Math 2A Quiz 1 Version 4
Fri Oct 7, 2016

NAME YOU ASKED TO BE CALLED IN CLASS: [redacted]

SCORE: _____ / 30 POINTS

1. NO CALCULATORS OR NOTES ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

Determine if $y = Ax + Be^{-2x} + \frac{x^2}{2}$ is a family of solutions of the DE $(2x+1)y'' + 4xy' - 4y = 4x^2 + 4x + 4$. **SCORE:** 6 **/ 6 PTS**

State your conclusion clearly.

DE: $(2x+1)y'' + 4xy' - 4y = 4x^2 + 4x + 4$

$y = Ax + Be^{-2x} + \frac{x^2}{2}$

$\frac{dy}{dx} = A + -2Be^{-2x} + x$ 1

$\frac{d^2y}{dx^2} = 4Be^{-2x} + 1$ 1

$(2x+1)(4Be^{-2x}+1) + (4x)(A - 2Be^{-2x} + x) - 4(Ax + Be^{-2x} + \frac{x^2}{2}) = 4x^2 + 4x + 4$

$\cancel{8xBe^{-2x}} + 2x + \cancel{4Be^{-2x}} + 1 + \cancel{4Ax} - \cancel{8xBe^{-2x}} + 4x^2 - \cancel{4Ax} - \cancel{4Be^{-2x}} - 2x^2 = 4x^2 + 4x + 4$

1

$2x^2 + 2x + 1 = 4x^2 + 4x + 4$

2 X No, 1 y is not a family of solutions to the DE.

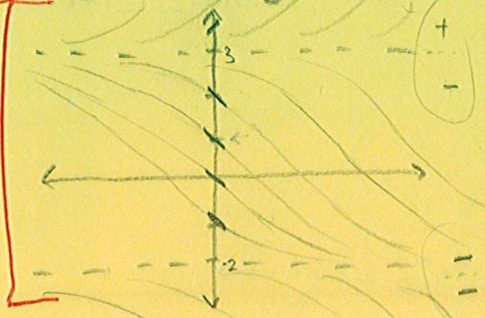
Consider the DE $\frac{dy}{dx} = (y^2 - y - 6)(y+2)$. **SCORE:** 6 **/ 6 PTS**

$(y-3)(y+2)(y+2)$

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
You must draw a phase portrait to get full credit.

1 $\frac{dy}{dx} = 0$ @ $y = -2, 3$

$\frac{dy}{dx}$ @ $y > 3 = (+)(+)(+) = +$
@ $-2 < y < 3 = (-)(+)(+) = -$
@ $y < -2 = (-)(-)(-) = -$

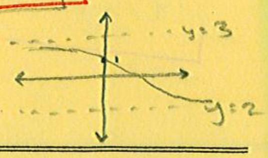


the eq. solutions are
1 $y = 3$ ← UNSTABLE 1
1 $y = -2$ ← SEMISTABLE 1

[b] If $y = m(x)$ is a solution of the DE such that $m(5) = 1$, what is $\lim_{x\to\infty} m(x)$?

$y = m(x)$
$y(5) = m(5) = 1$
so $m(x)$ is a curve that occupies the space between the asymptotes $y = 3$ & $y = -2$.

so... $\boxed{\lim_{x\to\infty} m(x) = -2}$ 1 (from above)



Consider the IVP $y' = 2xy^2 - 3x, \quad y(-1) = 2$. Use Euler's method with $h = 0.2$ to estimate $y(-0.6)$. **SCORE: 4 / 4 PTS**

$y(x+h) = y(x) + y'(x,y)(h)$

$y'(-1,2) = 2(-1)(2^2) - 3(-1)$

$y' = -8 + 3 = -5$

$y(-0.8) = y(-1) + y'(-1,2)(0.2)$

$y(-0.8) = 2 + (-5)(0.2)$ 1

$y(-0.8) = 2 - 1 = 1$ 1

$y'(-0.8,1) = 2(-0.8)(1)^2 - 3(-0.8)$

$y'(-0.8,1) = -1.6 + 2.4$

$y'(-0.8,1) = 0.8$

$y(-0.6) = y(-0.8) + y'(-0.8,1)(0.2)$

$y(-0.6) = 1 + (0.8)(0.2)$ 1 $\frac{8}{10} \cdot \frac{2}{10} = \frac{16}{100}$

$y(-0.6) = 1 + (0.16)$

$\boxed{y(-0.6) = 1.16}$ 1

In a certain society, the rate at which a person's wealth changes is proportional to the difference between their wealth and a fixed baseline (call it B, where $B > 0$). If everyone is getting poorer (except for those whose wealth equals the baseline), write a DE for the wealth of a person whose current wealth is half of the baseline. **SCORE: _____ / 4 PTS**

Justify the signs of all symbolic constants (other than B) in your DE properly, but briefly, as shown in lecture.

Do NOT use the absolute value function in your answer.

$W(t)$ = a person's wealth @ time t.

$\frac{dW}{dt}$: rate at which a person's wealth changes

$\frac{dW}{dt} = \underbrace{k}_{\text{proportional}}\underbrace{(W - B)}_{\text{difference between individuals wealth and a baseline.}}$ 2 where $B > 0$.

$\frac{dW}{dt} = -k(B - W)$ for all people.

to represent decreasing slope → W is always pos. as it is an amount of $

For someone w/ $W(t) = \frac{1}{2}B$,

$\frac{dW}{dt} = -k\left(B - \frac{1}{2}B\right)$

$\boxed{\frac{dW}{dt} = -k\left(\frac{B}{2}\right)}$ where $B > 0$.

(*)

→ For a person whose wealth is half of the baseline, $W - B$ is neg, $\frac{1}{2}$ and $\frac{dW}{dt}$ should be negative (as everyone poorer than the baseline is getting poorer) $\frac{1}{2}$

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP $(y')^3 - 1 = x + y, \quad y(1) = -2$? **Justify your answer properly, but briefly.** **SCORE: 4 / 4 PTS**

If f and f_y is cont. @ $y(1,-2)$ then there is a unique sol'n around $(1,-2)$

$y' = (x+y+1)^{1/3}$ 1

$f = (x+y+1)^{1/3}$ 1 → f @ $(1,-2) = (1-2+1)^{1/3} = (0)^{1/3} = 0$ ← continuous

$f_y = \frac{1}{3}(x+y+1)^{-2/3}(1)$ → f_y @ $(1,-2) = \frac{1}{3(1-2+1)^{2/3}} = \frac{1}{3(0)^{2/3}}$ X

not continuous. 1

So, the Existence & Uniqueness Theorem does not tell us if there is a unique sol'n around $(1,-2)$. However, it does not prove that there is NO solution around $(1,-2)$ 1